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CONCEPTS & COMPETENCE

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Exploding TENTH VOLCANO — MATHEMATICS HW

ANSWERS & SOLUTIONS

POLYNOMIALS

Ans.1 We have

$p(x) = 2x^2 - 15x + 25$

$= 2x^2 - (10 + 5)x + 25$

$= 2x^2 - 10x - 5x + 25$

$= (2x^2 - 10x) - (5x - 25)$

$= 2x(x - 5)(2x - 5)$

To find the zeroes of the polynomial p(x)

Put $p(x) = 0.$

$\Rightarrow$ $(x - 5)(2x - 5) = 0.$

$\Rightarrow$ $x - 5 = 0$

$\Rightarrow$ $x = 5$

Or $2x - 5 = 0$

$\Rightarrow$ $2x = 5$

$\Rightarrow$ $x = \frac{5}{2}$

$\therefore$ $x = 5, \frac{5}{2}$

Therefore zeroes of the polynomial p(x) are : $a = 5$ and $\beta = \frac{5}{2}$

Now, sum of the zeroes $= a + \beta = 5 + \frac{5}{2} = \frac{10+5}{2} = \frac{15}{2} = -\left(\frac{-15}{2}\right)$

$= -\frac{\text{coefficient of x}}{\text{coefficient of } x^2}$

Product of the zeroes $= a \times \beta = 5 \times \frac{5}{2} = \frac{25}{2} = \frac{\text{constant term}}{\textit{coefficient} \text{ of } x^2}$

$\therefore$ *Zeroes of the polynomial p(x) are* $5, \frac{5}{2}$.

Ans.2 If = a and β are the two zeroes of quadratic polynomial $x^2 + 2x + 35$.

$\Rightarrow$ $= a + \beta = -\frac{b}{a} = -\frac{2}{1} = -2,$ ... (i)

and $a \times \beta = \frac{c}{a} = -\frac{35}{1} = -35$. … (ii)

Now, we have the quadratic polynomial $ax^2 + bx + c$, *whose zeroes are* $\frac{1}{a}$ and $\frac{1}{\beta}$.

$\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{b}{a}$

$\Rightarrow$ $\frac{\beta + a}{a\beta} = -\frac{b}{a}$

$\Rightarrow$ $\frac{a + \beta}{a\beta} = -\frac{b}{a}$

$\Rightarrow$ $\frac{a + \beta}{a\beta} = -\frac{b}{a}$ [From (i) and (ii), $a + \beta = -2$, $a \times \beta = -35$]

Sum of zeroes = $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{2}{35}$

Product of zeroes = $\frac{1}{\alpha}\times\frac{1}{\beta}=-\frac{1}{35}$

Poly = $x^2 - Sx + P$

$= x^2 - \frac{2}{35}x - \frac{1}{35}$

$= 35\left(x^2 - \frac{2x}{35} - \frac{1}{35}\right)$

$= 35x^2 - 2x - 1$ *Ans.*

Ans.3 We have

$f(x) = x^2 - 7x + p$

$\because a$ and β are the zeroes of the polynomial $f(x) = x^2 - 7x + p$.

$\therefore \quad a + \beta = -\frac{b}{a} = \frac{-(7)}{1} = 7$

and $\quad a \times \beta = \frac{c}{a} = \frac{p}{1} = p$

Now $\quad a^2 + \beta^2 = 29$ [Adding and subtracting $2\alpha\beta$]

$\Rightarrow \quad a^2 + \beta^2 + 2a\beta - 2a\beta = 29$

$\Rightarrow \quad (a + \beta)^2 - 2a\beta = 29$

$\Rightarrow \quad (7)^2 - 2 \times p = 29$

$\Rightarrow \quad 49 - 2p = 29$

$\Rightarrow \quad 2p = 49 - 29 = 20$

$\Rightarrow \quad p = \frac{20}{2} = 10.$

Ans. 4 We have

Sum of zeroes = $(a + \beta) = -3$

Product of zeroes = $(a \times \beta) = 2$

$\therefore$ Quadratic polynomial is $p(x) = [x^2 - (a + \beta)\,x + a \times \beta]$

$= x^2 - (-3)\,x + 2$

$= x^2 + 3x + 2.$

Ans. 5 $\frac{1}{3}$

Ans. 6 Let the $\alpha + \beta + \gamma$ are zeroes of cubic polynomial f(x)

According to question, we have

$\alpha + \beta + \gamma = 3.$

$\alpha\beta + \beta\gamma + \gamma\alpha = 4.$

and $\quad \alpha \times \beta \times \gamma = -6$

Then cubic polynomial is: $f(x) = x^3 - (\alpha + \beta + \gamma)\,x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)\,x - \alpha\beta\gamma$

$= x^3 - 3x^2 + 4x + 6.$

Ans.7 Since α, β are zeroes of the polynomial

$p(x) = x^2 + x - 12$ [Here, $a = 1, b = 1, c = -2$]

Sum of zeroes = $a + \beta = -\frac{b}{a}$

$= -\frac{1}{1}$

$= -1$

and, $\quad$ Product of zeroes = $a \times \beta = \frac{c}{a}$

$= -\frac{12}{1}$

$= -12$

(i) Now $a^2 + \beta^2 = a^2 + \beta^2 + 2a\beta - 2a\beta$

$= (a + \beta)^2 - 2a\beta$

$= (-1)^2 - 2\times -12$ [Since $a + \beta = -1$ and $a \times \beta = -12$]

$= 1 + 24 = 25$

(ii) $\frac{1}{a} + \frac{1}{\beta} - 2\alpha\beta = \frac{\alpha+\beta}{a\beta} - 2\alpha\beta$

$= \frac{-1}{-12} - 2\times(-12)$ [Since $a + \beta = -1$ and $a \times \beta = -12$]

$= \frac{1}{12} + 24$

$= \frac{1+288}{12} = \frac{289}{12}$.

(iii) $a^3 + \beta^3 = (a + \beta)^3 - 3a\beta(a + b)$

$= (-1)^3 - 3 \times -12 \times (-1)$ [Since $\alpha + \beta = -1$ and $\alpha \times \beta = -12$]

$= -1 + 36 \times -1$

$= -1 - 36$

$= -37$

Ans. 8 Since the α and β are zeroes of the polynomial.

$p(x) = kx^2 + 5x + 2$ [Here, $a = k$, $b = 5$, $c = 2$]

$\alpha + \beta = -\frac{b}{a}$

$= -\frac{5}{k}$

and $\alpha \times \beta = \frac{c}{a} = \frac{2}{k}$

According to question,

$$\frac{1}{a^2 +} + \frac{1}{\beta^2} = \frac{17}{4}$$

$$\Rightarrow \frac{\alpha^2+\beta^2}{a^2\beta^2} = \frac{17}{4}$$

$$\Rightarrow \frac{a^2+\beta^2+2\alpha\beta-2\alpha\beta}{a^2\beta^2} = \frac{17}{4}$$

$$\Rightarrow \frac{(\alpha+\beta\}^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{17}{4}$$

$\Rightarrow$

$$\Rightarrow \frac{\left(-\frac{5}{k}\right)^2 - 2\times\frac{2}{k}}{\left(\frac{2}{k}\right)^2} = \frac{17}{4}$$

$$\Rightarrow \frac{\frac{25}{k^2} - \frac{4}{k}}{\frac{4}{k^2}} = \frac{17}{4}$$ $\left[\begin{array}{l}\text{Since, } \alpha + \beta = -\frac{5}{k} \\ \text{and } \alpha \times \beta = \frac{2}{k}\end{array}\right]$

$$\Rightarrow \frac{\frac{25-4k}{k^2}}{\frac{4}{k^2}} = \frac{17}{4}$$

$$\Rightarrow \frac{25-4k}{k^2} \times \frac{k^2}{4} = \frac{17}{4}$$

$\Rightarrow \quad 25 - 4k = \dfrac{4 \times 17}{4}$

$\Rightarrow \quad 25 - 4k = 17$

$\Rightarrow \quad 4k - 25 = 17.$

$\Rightarrow \quad 4k = 8$

$\Rightarrow \quad k = 2.$

Ans. 9 $x - 1 \overline{)\, 4x^3 - 3x^2 + 2x - 4 \,(} \, 4x^2 + x + 3$

$4x^3 - 4x^2$

$- \quad +$

$x - 1 \overline{)\, 4x^3 - 3x^2 + 2x - 4 \,(} \, 4x^2 + x + 3$

$4x^3 - 4x^2$

$- \quad +$

$3x - 4$

$3x - 3x$

$- \quad +$

-1

Here quotient $q(x) = 4x^2 + x + 3$ and remainder $r(x) = -1$

$\because f(x) = g(x) \times q(x) + r(x)$

$\therefore 4x^3 - 3x^2 - 4 = (x - 1)(4x^2 + x + 3) + (-1)$

Ans. 10 We know that if a is zero of the polynomial then $(x - a)$ *is a factor of given polynomial.*

Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$. are zeroes of the polynomial $f(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

Hence, $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ is a factor of $f(x)$.

Now, we divide $f(x)$ by $x^2 - \frac{5}{3}$ to obtain the other zeroes as follows:

$x^2 - \frac{5}{3} \overline{)\, 3x^4 + 6x^3 - 2x^2 - 10x - 5 \,(} \, 3x^2 + 6x + 3$

$3x^4 \quad - 5x^2$

$- \quad +$

$6x^3 + 3x^2 - 10x - 5$

$6x^3 \quad - 10x$

$- \quad +$

$3x^2 - 5$

$3x^2 - 5$

$- \quad +$

0

According to division algorithm of polynomial

$$\therefore \quad 3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$$

$$= \left(x^2 - \frac{5}{3}\right) \times 3(x^2 + 2x + 1)$$

$$= 3\left(x^2 - \frac{5}{3}\right)(x + 1)^2$$

For zeroes of the polynomial $f(x) = 0$

$$\therefore \quad 3\left(x^2 - \frac{5}{3}\right)(x + 1)^2 = 0$$

$$\Rightarrow \quad \left(x + \sqrt{\frac{5}{3}}\right)\left(x - \sqrt{\frac{5}{3}}\right)(x + 1)(x + 1) = 0$$

$$\Rightarrow \quad x = -\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}, -1, -1$$

Hence, the other zeroes of polynomial are $-1, -1$.