ANSWERS & SOLUTIONS

REAL NUMBERS

Ans. 1 Let us assume, that $\sqrt{5}$ is rational. It can be express in the form of $\frac{a}{b}$, where a and b are coprime positive integers and b $\neq 0$. $\sqrt{5} = \frac{a}{b}$ (where a and b are coprime $\therefore$ HCF of a and b is 1)

Squaring both sides $5 = \frac{a^2}{b^2}$

$\Rightarrow$ $5b^2 = a^2$ … (i)

Therefore, 5 divides a^2. It follows that 5 divides a [By theorem 1.3]

Let $a = 5c$ and put this value in equation (i) [Where c is any positive integer]

$5b^2 = (5c)^2$

$\Rightarrow$ $5b^2 = 25\,c^2$

$\Rightarrow$ $\frac{25}{5}c^2 = b^2$

$\Rightarrow$ $5c^2 = b^2$

It means b^2 is divisible by 5. It follows that b, is divisible by 5.

Ans. 2 Let us assume, that $3+5\sqrt{2}$ is rational. It can be express in the form of $\frac{a}{b}$, where a, b are coprime positive integers and b $\neq 0$.

$\therefore$ $3+5\sqrt{2} = \frac{a}{b}$ (Where HCF of a and b = 1)

$\Rightarrow$ $5\,5\sqrt{2} = \frac{a}{b} - 3.$

$\Rightarrow$ $5\sqrt{2} = \frac{a-3b}{b}$

$\Rightarrow$ $\sqrt{2} = \frac{a-3b}{5b}$

$\because$ a and b are positive integer.

$\therefore$ $\frac{a-3b}{5b}$ is rational.

Therefore, $\sqrt{2}$ is rational. But this contradicts the fact $\sqrt{2}$ is rational. So, our assumption $3 + 5\sqrt{2}$ is rational is wrong.

Hence, $3 + 5\sqrt{2}$ is an irrational number.

Ans. 3 Let x be any given positive integer and b = 3. There exists positive integer q and r such that

$x = 3q + r$, where $0 \leq r < 3$

$x = 3q$, $3q + 1$ or $3q + 2$.

So we have the following cases:

Case I: When $x = 3q$ $x^2 = 9q^2 = 3\,(3q^2)$ [Squaring both sides]

$x^2 = 3m$, where $m = 3q^2$

Case II: When $x = 3q + 1$ $x^2 =)3q + 1)^2$ [Squaring both sides]

$\Rightarrow$ $x^2 = 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1$

$= 3m + 1$, where $m = 3q^2 + 2q$

Case III: When $x = 3q + 2$ $x^2 = (3q + 2)^2$ [Squaring both sides]

$= 9q^2 + 12q + 4$

$= 9q^2 + 12q + 3 + 1$

$= 3(3q^2 + 4q + 1) + 1$

$= 3m + 1$, where $m = 3q^2 + 4q + 1$

Hence, square of any positive integer is either of the form 3m or 3m + 1for some integer m.

Ans. 4 Let x be positive integer by apply Euclid's division lemma it is of the form 3q or 3q + 1 or 3q + 2.

So, we have the following cases:

Case I: When x = 3q.

$x^3 = (3q)^3 = 27q^3$ [Cube both sides]

$= 9(3q^3)$

$= 9m$, where $m = 3q^3$

Case II: when x = 3q + 1.

$x^3 = (3q + 1)^3$ [Cube both sides]

$= 27q^3 + 27q^2 + 9q + 1$

$= 9\ (3q^3 + 3q^2 + q) + 1$

$= 9m + 1$, where $m = 3q^3 + 3q^2 + q$

Case III: When x = 3q + 2.

$x^3 = (3q + 2)^3$

$= 27q3 + 54q^2 + 36q + 8$

$= 9q\ (3q^2 + 6q + 4) + 8$

$= 9m + 8$, where $m = q\ (3q^2 + 6q + 4)$

Hence, x^2 is the either of the form 9m, 9m + 1 or 9m + 8.

Ans. 5 If the number 4^n ends with the digit zero, then it is divisible by 5, hence, it has 5 as prime factor

But $4^n = (2 \times 2)^n = (2^2)^n = 2^{2n}$.

i.e. 4^n contain prime 2 only.

Uniqueness of fundamental theorem of arithmetic guarantees that there are no other primes is the factorization of 4^n

$\Rightarrow 4^n$ does not contain prime 5.

Hence, 4^n can not end with the digit 0 for any natural number n. **Ans.**

Ans. 6 (i) 2.535353…. It is the form of non terminating but recurring decimal. So, it is rational.

(ii) **5.**060060006…

It is the form of non terminating non recurring decimal**.** So, it is irrational.

(iii) $\frac{7}{8}$, $\frac{7}{8} = 0.875$, it is the form of terminating decimal. So it is rational.

(iv) $\frac{49}{500}$. $\frac{49}{500} = 0.098$

It is the form of terminating decimal. So, it is rational.

(v) $\frac{22}{7}$. $\frac{22}{7} = \overline{3.142857}$

It is of the form of non terminating but recurring decimal. So, it is rational.

(vi) $\overline{0.1428571}$ It is of the form of non terminating but recurring decimal. So it is rational.

(v) π. π. Is irrational

Ans. 7 (i) $\frac{49}{2^2 5^3}$, Since denominator of $\frac{49}{2^2 5^3}$ is of the form $2^m\ 5^n$, where m and n are non negative integers. Therefore $\frac{49}{2^2 5^3}$ has terminating decimal expansion

(ii) $\frac{32}{450}$**,** $\frac{32}{450} = \frac{2\times2\times2\times2\times2}{2\times3\times3\times5\times5} = \frac{16}{3^2\times5^2}$

Since the denominator of $\frac{32}{450}$ is not of the form $2^m\ 5^n$. $\frac{32}{450}$ Hence has non terminating repeating decimal expansion.

(iii) $\frac{237}{2^2 3^2 5^3}$

Since the denominator of $\frac{237}{2^2 3^2 5^3}$ is not of the form 2^m, 5^n. Hence $\frac{237}{2^2 3^2 5^3}$ has non terminating repeating decimal expansion.

(iv) $\frac{150}{441}$

$$\frac{150}{441} = \frac{2\times3\times5\times5}{3\times3\times7\times7} = \frac{2\times5\times5}{3\times7\times7} = \frac{50}{3\times7^2}$$

Since the denominator of $\frac{150}{441}$ is not of the form of $2^m\ 5^n$. Hence, it has non terminating repeating decimal expansion

(v) $\frac{85}{8}$

$$\frac{85}{8} = \frac{85}{2^3} = \frac{85}{2^3 \times 5^0}$$

Since the denominator of $\frac{85}{8}$ is of the form of $2^m\ 5^n$, where m and n are non negative integers. Hence, it has terminating decimal expansion.

(vi) $\frac{256}{625}$

$$\frac{256}{625} = \frac{256}{5 \times 5 \times 5 \times 5} = \frac{256}{5^4} = \frac{256}{2^0 \times 5^4}$$

Since the denominator of $\frac{256}{625}$ is of the form of $2^m\ 5^n$, where m and n are non – negative integers. Hence, it has terminating decimal expansion.

Ans. 8 **(ii)**

Ans: 9 **(i)**

Ans: 10 **(ii)**