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CONCEPTS & COMPETENCE
A Solution... to get you Ahead !

PRE-FINAL ASSESMENT
Synchro - School Program

Foundation - IX & SPINE
Special Preliminary Initiative for National Level Exams

P - F - A

A Foundation Stone for NTSE / STSE / IJSO / Kishore Vaigyanik / M-Sc Olympiads / IIT / PMT / MBA / IAS

PRE-FINALS MATHEMATICS - 2 (Std. IX)

FULL SYLLABUS SOLUTION KEY **M.M: 80**

General Instructions:

1. The question paper comprises of four sections, A, B, C & D.
2. All questions are compulsory.
3. Section A Q1 to 6 are 1 mark each
4. Section B Q7 to 12 are 2 marks each.
5. Section C Q13 to 22 are 3 marks each.
6. Section D Q23 to 30 are 4 marks each.
7. Do proper numbering of Answer in copy & Draw Neat Diagrams
8. Attempt All sections & Q's in sequential order

SECTION – A

Ans 1. AXIOMS: The basic facts which are taken for granted, without proof, are called axioms.
THEOREMS: The conclusions obtained through logical reasoning based on previously proved results and some axioms constitute a statement which is known as a theorem or a proposition.

Ans 2. We have $p(x) = 7x^3(x^2 - 5) = 7x^5 - 35x^3$
Cleary, the degree of $p(x) = 5$.

Ans 3. We know that the volume V of a right circular cone of radius r and height h is given by

$$V = \frac{1}{3}\pi r^2 h$$

Here, $r = 28$ cm and $h = 1.02$ m $= 102$ cm

$$\therefore \quad V = \frac{1}{3} \times \frac{22}{7} \times 28 \times 28 \times 102 \text{ cm} = 83776 \text{ cm}^3$$

Ans 4. Abscissa of (−2, 7) is −2.

Ans 5. $(125)^{-1/3} = \dfrac{1}{(125)^{1/3}} = \dfrac{1}{(5^3)^{1/3}} = \dfrac{1}{5^{3\times\frac{1}{3}}} = \dfrac{1}{5}$

Ans6. Since $ABCD$ is a cyclic quadrilateral.

$\therefore \quad \angle BCD + \angle BAD = 180°$

$\Rightarrow \quad \angle BCD + 70° = 180°$

$\Rightarrow \quad \angle BCD = 110°$

In ΔBCD, we have

$\angle CBD + \angle BCD + \angle BDC = 180°$

$\Rightarrow \quad 30° + 110° + \angle BDC = 180°$

$\Rightarrow \quad \angle BDC = 40°$

Since LADB is the angle in a semi-circle.

$\therefore \quad \angle ADB = 90°$

In Δ ABD, we have

$\angle ABD + \angle ADB + \angle BAD = 180°$

$\Rightarrow \quad \angle ABD + 90° + 70° = 180°$

$\Rightarrow \quad \angle ABD = 20°$

Hence, $\angle ABD = 20°$ and $\angle BDC = 40°$

Ans 7. **GIVEN**: A trapezium ABCD in which AB II DC, E is the mid-point of AD and F is a point on BC such that EF||DC.

TO PROVE EF = $\frac{1}{2}$(AB+ DC)

PROOF In ΔADC, E is the mid-point of AD and EG||DC (Given)

∴ G is the mid-point of AC

Since segment joining the mid-points of two sides of a triangle is half of the third side.

∴ EG = $\frac{1}{2}$ DC ... (i)

Now, ABCD is a trapezium in which AB||DC.

But, EF||DC

∴ EF||AB

⇒ GF||AB

In ΔABC, G is the mid-point of AC (proved above) and GF||AB.

∴ F is the mid-point of BC

⇒ GF = $\frac{1}{2}$ AB ... (ii) [∵ Segment joining the mid-point of two sides of a Δ is half of the third side]

From (i) and (ii), we have

EG + GF = $\frac{1}{2}$(DC) + $\frac{1}{2}$(AB) ⇒ EF = $\frac{1}{2}$ (AB + DC)

Ans 8. (2,6) The perpendicular distance of a point from the *x-axis* is its ordinate so, the ordinate of the point is 6. Similarity, the perpendicular distance the *y-axis* is its abscissa so, the abscises of the point is 2.

Ans 9. Consider the arc BD. Clearly, it makes an angle $\angle$BOD = 140° at the centre O of the circle and $\angle$BAD at a point A in the circle

∴ $\angle BAD = \frac{1}{2}\angle BOD = 70°$

Since ABCD is a cyclic quadrilateral.

∴ $\angle BAD + \angle BCD = 180°$

⇒ $70° + \angle BCD = 180°$

⇒ $\angle BCD = 110°$

Now, $\angle BCD + \angle DCP = 180°$ [∵ $\angle$BCD and $\angle$DCP are linear pairs]

⇒ $110° + \angle DCP = 180°$

⇒ $\angle DCP = 70°$

Hence, $\angle BAD = \angle DCP = 70°$

Ans10. Since the dimensions of the cuboid are in the ratio 1 : 2 : 3. So, let the dimensions x, $2x$, $3x$ in metres. Now,

Surface area = 88 m^2

⇒ $2(x \times 2x + 2x \times 3x + x \times 3x) = 88$

⇒ $2(2x^2 + 6x^2 + 3x^2) = 88$

⇒ $2 \times 11x^2 = 88$

⇒ $22x^2 = 88$ ⇒ $x^2 = \frac{88}{22}$ ⇒ $x^2 = 4$ ⇒ $x^2 = 2^2$ ⇒ $x = 2$ m

⇒ $2x = 2 \times 2 = 4$ and $3x = 3 \times 2 = 6$.

Hence, the dimensions are 2 m, 4 m and 6 m.

Ans 11. Calculation of Mean.

x_i	f_i	$f_i\,x_i$
3	6	18
5	8	40
7	15	105
9	p	$9p$
11	8	88
13	4	52
	N = $\sum f_i = 41 + p$	$\sum f_i x_i = 303 + 9p$

We have, $\sum f_i = 41 + p,\ \sum f_i x_i = 303 + 9p$

$\therefore$ Mean $= \dfrac{\sum f_i x_i}{\sum f_i} \Rightarrow 7.5 = \dfrac{303+9p}{41+p}$

$\Rightarrow 7.5 \times (41 + p) = 303 + 9p \Rightarrow 307.5 + 7.5p = 303 + 9p$

$\Rightarrow 9p - 7.5p = 307.5 - 303 \Rightarrow 1.5p = 4.5 \Rightarrow p = 3$

Ans 12. We have, $\dfrac{2+\sqrt{3}}{2-\sqrt{3}} + \dfrac{2-\sqrt{3}}{2+\sqrt{3}} = \dfrac{\left(2+\sqrt{3}\right)^2 + \left(2-\sqrt{3}\right)^2}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}$

$= \dfrac{4+3+4\sqrt{3}+4+3-4\sqrt{3}}{4-3} = \dfrac{14}{1} = 14.$

Ans 13. $\left[x - \dfrac{2}{3}y\right]^3 = x^3 - \left(\dfrac{2}{3}y^3\right) - 3(x)\left(\dfrac{2}{3}y\right)\left(x - \dfrac{2}{3}y\right)$

$= x^3 \dfrac{8}{27}y^3 - 2xy\left(x - \dfrac{2}{3}y\right) = x^3 - \dfrac{8}{27}y^3 - 2x^2y + \dfrac{4}{3}xy^2$

Ans14. We know that, if *AB* and *AC* are two equal chords of a circle, and OA = OB = 5 cm.

Here, *AB* = *AC* = 6 cm.

$(ar)\ \Delta$ OBA = $\sqrt{8(8-5)(8-5)(8-6)}$

$= \sqrt{8\times3\times3\times2}$

$(ar)\ \Delta$ OBA = 12

(ar) OBA $= \dfrac{1}{2}\times\text{OA}\times\text{BM}$

$12 = \dfrac{1}{2} \times 5 \times \text{BM}$

$12 \times \dfrac{1}{2} \times \dfrac{1}{5} = \text{BM}$

BM = 4.8   BC = 4.8 × 2 = 9.6

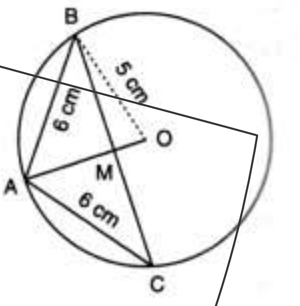


Fig for Q.14

Ans 15. We know that $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ … (i)

Here, $a = 3x$, $b = y$ and $c = z$

Now, writing the given algebraic expression in the form of the identify (i) we have,

$(3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z) = (3x + y + z)[(3x)^2 + y^2 + z^2 - 3x(y) - yz - z(3x)]$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - zx)$

Hence, $27x^3 + y^3 + z^3 - 9xyz = (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - zx)$

Ans16. Since A APB and parallelogram ABCD are on the same base AB and between the same parallels.

$\therefore \quad ar\,(\Delta APB) = \frac{1}{2}\, ar\,(\|^{gm}\ ABCD) \qquad ...(i)$

Similarly,

$ar\,(\Delta BQC) = \frac{1}{2}\, ar\,(\|^{gm}\ ABCD) \qquad ...(ii)$

From (i) and (ii), we have

$ar\,(\Delta\ APB) = ar\,(BQC)$

Ans17. Let A denote the event that the batsman did not hit a boundary.

We have, Total number of trials = 40

Number of trials in which the event A happened = 40 – 8 = 32

$\therefore \quad P(A) = \frac{32}{40} = \frac{4}{5} = 0.8$

Ans 18. Let the radius r and slant height *h* of the cone be 3*x* cm and 4*x* cm respectively.

Then,

Volume = 301.44 cm^3

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 301.44\ cm^3$

$\Rightarrow \quad \frac{1}{3} \times 1 \times 3.14 \times 3x \times 3x \times 4x = 301.44$

$\Rightarrow \quad x^3 = \frac{301.44}{3 \times 4 \times 3.14} \Rightarrow x^3 = \frac{301.44}{37.68} = 8 \Rightarrow x = 2$

$\therefore \quad$ r = radius = 3*x* = 6 cm and *h* = height = 4*x* = 8 cm

Now, Slant height = $\sqrt{r^2 + h^2}$ cm = $\sqrt{36 + 64}$ cm = 10 cm

Ans 19. (2) (−4, 0)

Ans 20. Join AC.

In ΔABC, BC > AB [∵ AB is shortest]

$\Rightarrow \quad \angle BAC > \angle BCA$ … (i)

[Angle opposite to longer side is greater]

In ΔACD, DC > AD [∵ AB is longest]

$\Rightarrow \quad \angle CAD > \angle ACD$ … (ii)

[Angle opposite to longer side is greater]

Adding (i) and (ii), we have

$\angle BAC + \angle BCA > \angle BCA > \angle ACD$

$\Rightarrow \quad \angle A > \angle C.$ **Proved.**

Ans 21. We have,

$(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$

$\therefore \quad (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a - b)\,(a + b)\,(b - c)\,(b + c)\,(c - a)\,(c + a)$

Similarly, we have,

$(a - b) + (b - c) + (c - a) = 0$

$\Rightarrow \quad (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$

$\therefore \quad \frac{(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$

$= \frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)c-a)} = (a + b)\,(b + c)\,(c + a)$

Ans22. We observe that the triangles BAC and EAC are on the same base AC and between the same parallels AC and BE.

$\therefore$ *ar* (ΔBAC) = *ar* (ΔEAC)

$\Rightarrow$ *ar* (ΔBAC)+ *ar* (ΔADC) = *ar* (ΔEAC) + *ar* (ΔADC) [Adding *ar* (ΔADC) on both sides]

$\Rightarrow$ *ar* (quad. ABCD) = *ar* (ΔADE)

23. Find the values of x and y, if $(x + 5, 3y - 5) = (8, 1)$

Ans 23. By definition of equality of two ordered pairs, we have

$(x + 5, 3y - 5) = (8. 1)$

$\Rightarrow$ $x + 5 = 8$ and $3y - 5 = 1$

$\Rightarrow$ $x = 8 - 5$ and $3y = 1 + 5$

$\Rightarrow$ $x = 3$ and $3y = 6$

$\Rightarrow$ $x = 3$ and $y = \frac{6}{3} = 2$

Hence, $x = 3$ and $y = 2$

Ans24. Since X and Y are the mid-points of AB and DC respectively.

$\therefore$ AX = $\frac{1}{2}AB$ and $CY = \frac{1}{2}DC$ ... (i)

But, AB = DC [$\because$ ABCD is a ||^{gm}]

$\Rightarrow$ $\frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow$ AX = CY ... (ii)

Also, AB ||DC

$\Rightarrow$ AX ||YC ... (iii)

Thus, in quadrilateral AXCY, we have

AX ||YC and AX = YC [From (ii) and (iii)]

Hence, quadrilateral AXCY is a parallelogram.

Ans 25. In order to factorize $x^2 + 3\sqrt{3}\ x - 30$, we have to find two numbers p and q such that $p + q = 3\sqrt{3x} - 30$ and $pq = - 30$.

Clearly, $5\sqrt{3} + (-2\sqrt{3}) = 3\sqrt{3}$ and $5\sqrt{3} \times 2\sqrt{3} = -30$

So, we write the middle term $3\sqrt{3}x$ as $5\sqrt{3}x - 2\sqrt{3}x$

$\therefore$ $x^2 + 3\sqrt{3x} - 30$

$= x2 + 5\sqrt{3}x - 2\sqrt{3}x - 30$

$= \left(x^2 + 5\sqrt{3}x\right) - \left(2\sqrt{3}x + 30\right)$

$= \left(x^2 + 5\sqrt{3}x\right) - \left(2\sqrt{3}x - 10\sqrt{3} \times \sqrt{3}\right)$

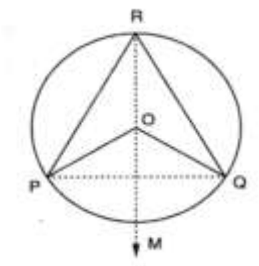
$= x(x + 5\sqrt{3}) - 2\sqrt{3}(x + 5\sqrt{3}) = (x + 5\sqrt{3})(x - 2\sqrt{3})$

Ans26. GIVEN An arc PQ of a circle C(0, r) and a point R on the remaining part of the circle i.e. arc QP.

TO PROVE $\angle$POQ = 2$\angle$PRQ

CONSTRUCTION Join RO and produce it to a point M outside the circle.

PROOF we shall consider the following three different cases:



CASE I When $\overset{\frown}{PQ}$ is a minor *arc* (see Fig. (*i*))

We know that an exterior angle of a triangle is equal to the sum of the interior opposite angles.

In ΔPOR, $\angle POM$ is the exterior angle.

$\therefore \quad \angle POM = \angle OPR + \angle ORP$

$\Rightarrow \quad \angle POM = \angle ORP + \angle ORP \qquad [\because OP = OR = r \therefore \angle OPR = \angle ORP]$

$\Rightarrow \quad \angle POM = 2\angle ORP \qquad$ ...(i)

In ΔQOR, $\angle QOM$ is the exterior angle.

$\therefore \quad \angle QOM = \angle OQR + \angle ORQ$

$\Rightarrow \quad \angle QOM = \angle ORQ + \angle ORQ \qquad [\because OQ = OR = r \therefore \angle ORQ = \angle OQR]$

$\Rightarrow \quad \angle QOM = 2\angle ORQ \qquad$ ...(ii)

Ading equations (i) and (ii), we get

$\angle POM + \angle QOM = 2\angle ORP + 2\angle ORQ$

$\Rightarrow \quad \angle POM + \angle QOM = 2(\angle ORP + \angle ORQ)$

$\Rightarrow \quad \angle POQ = 2\angle PRQ$

Ans 27. In ΔTQR,

$90° + 40° + x = 180° \qquad$ [Angle sum property of *a* Δ]

$\Rightarrow \quad 130° + x = 180°$

$\Rightarrow \quad x = 180° - 130°$

$\Rightarrow \quad x = 50°$

Now, $\quad y = \angle SPR + x \qquad$ [Exterior angle theorem]

$\Rightarrow \quad y = 30° + 50°$

$\Rightarrow \quad y = 80°$

Ans 28. Let ABCD be given quadrilateral shaped park such that

$\angle ABC = 90°$ and $AB = 9$ m, $BC = 40$ m

$CD = 15$ m

Join A to C

Now, in right ΔABC, we have

$AC^2 = AB2 + BC^2 \qquad$ (Using Pythagoras Theorem)

$\Rightarrow \quad AC^2 = (9)^2 + (40)^2 \Rightarrow AC^2 = 81 + 1600$

$\Rightarrow \quad AC^2 = 1681 \Rightarrow AC = \sqrt{1681}$

$\Rightarrow \quad AC = 41$ m

Now, area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

Area of ΔABC:

Here, $\quad AB = 9$ m, $BC = 40$ m and $AC = 41$ m

Semi-perimeter, $\quad s = \dfrac{AB + BC + AC}{2} = \dfrac{9m + 40m + 41m}{2} = \dfrac{90}{2} = 45m$

Using Heron's formula

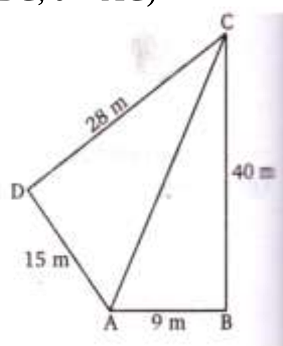
Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} \qquad (\because a = AB, b = BC, c = AC)$

$\Rightarrow \quad$ Area of $\Delta ABC = \sqrt{45(45-9)(45-40)(45-41)}$ m^2

$= \sqrt{45(36)(5)(4)}$ m^2

$= \sqrt{9 \times 5 \times 9 \times 4 \times 5 \times 4}$ m^2

$= (9\times5\times4)$ m^2 $= 180$ m^2



Area of ACD,

Here, $\quad AC = 41$ m, $CD = 28$ and $DA = 15$ m

Semi-perimeter, $\quad s = \dfrac{AC + CD + DA}{2} = \dfrac{41m + 28m + 15m}{2} = \dfrac{84}{2} = 42m$

Using Heron's formula

Area of ΔACD= $\sqrt{s(s-a)(s-b)(s-c)}$ (AC =a; CD = b; DA = c)

$\Rightarrow$ Area of ΔACD= $\sqrt{42(42-41)(42-28)(42-15)}$ m^2

$\Rightarrow$ $= \sqrt{42\times1\times14\times27} = \sqrt{14\times3\times14\times3\times3\times3}$ m^2

$= (14\times3\times3)\ m^2 = 126\ m^2$

Area of quadrilateral ABCD

$= (\text{Area of } \Delta ABC) + (\text{Area of } \Delta ACD) = (180 + 126)\ m^2$

$= 306\ m^2$

Ans29. Let the base radius of the cylindrical vessel be r and its height be h.

We have,

Circumference of the base = 132 cm and Height = 25 cm

$\therefore$ $2\pi r = 132$ and $h = 25$

$\Rightarrow$ $2\times\frac{22}{7}\times r = 132$

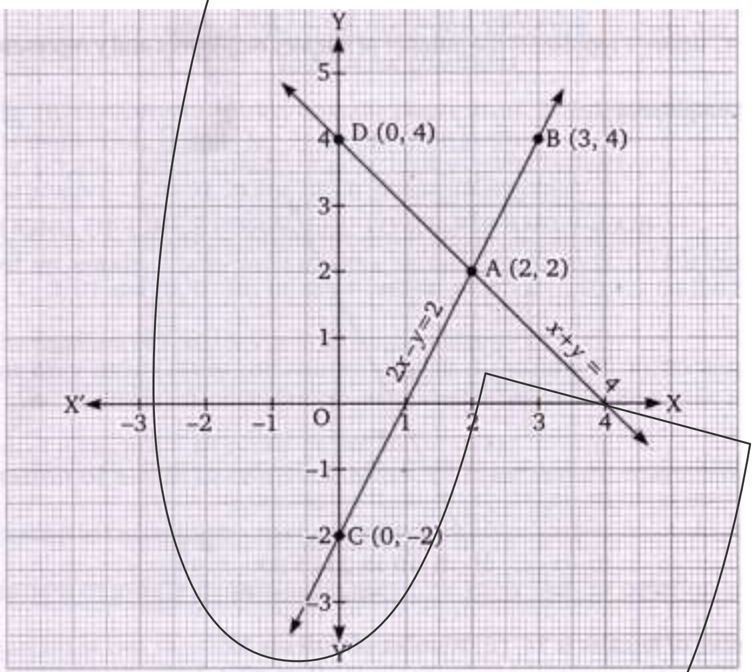
$\Rightarrow$ $r = \frac{132\times7}{2\times22} = 21$ and $h = 25$

$\therefore$ Volume of the vessel $= \pi r^2 h$

$= \frac{22}{7}\times(21)^2\times25cm^3 = 34650\ cm^3$

$= \frac{34650}{1000}$ liters = 34.65 litres [$\because$ 1000 cm^3 = 1 litres]

Ans 30.



Coordinates of point of intersection are (2, 2). So $x = 2$ and $y = 2$